



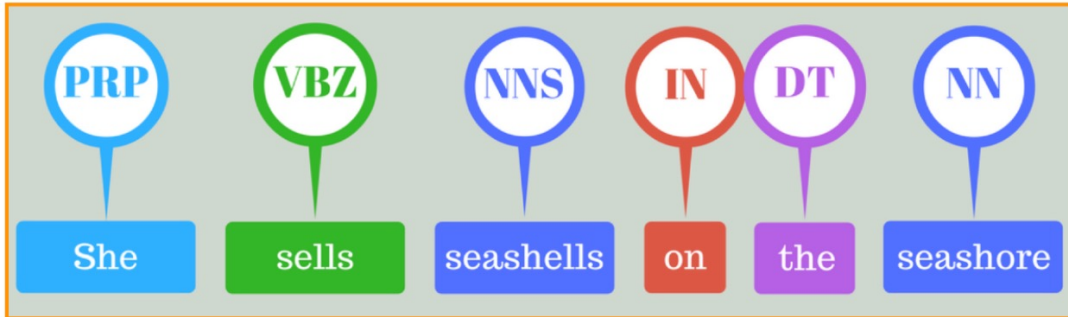
**CSE 4392 SPECIAL TOPICS
NATURAL LANGUAGE PROCESSING**

Sequence Models

1

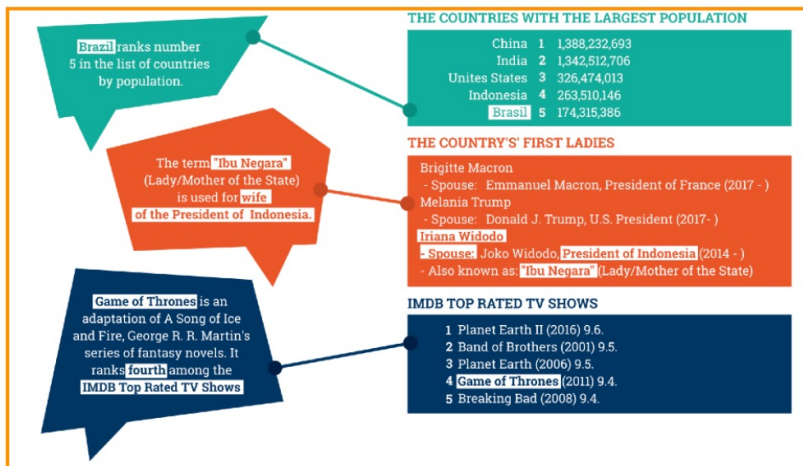
2024 Spring

WHY MODEL SEQUENCES?



Part-of-speech tagging

Name Entity Recognition



Information extraction

OVERVIEW

- Hidden Markov Models (HMM)
- Viterbi algorithm
- Conditional Random Field (CRF)

WHAT ARE POS TAGS?

- Word classes or syntactic categories
 - Reveal useful information about a word (and its neighbors!)

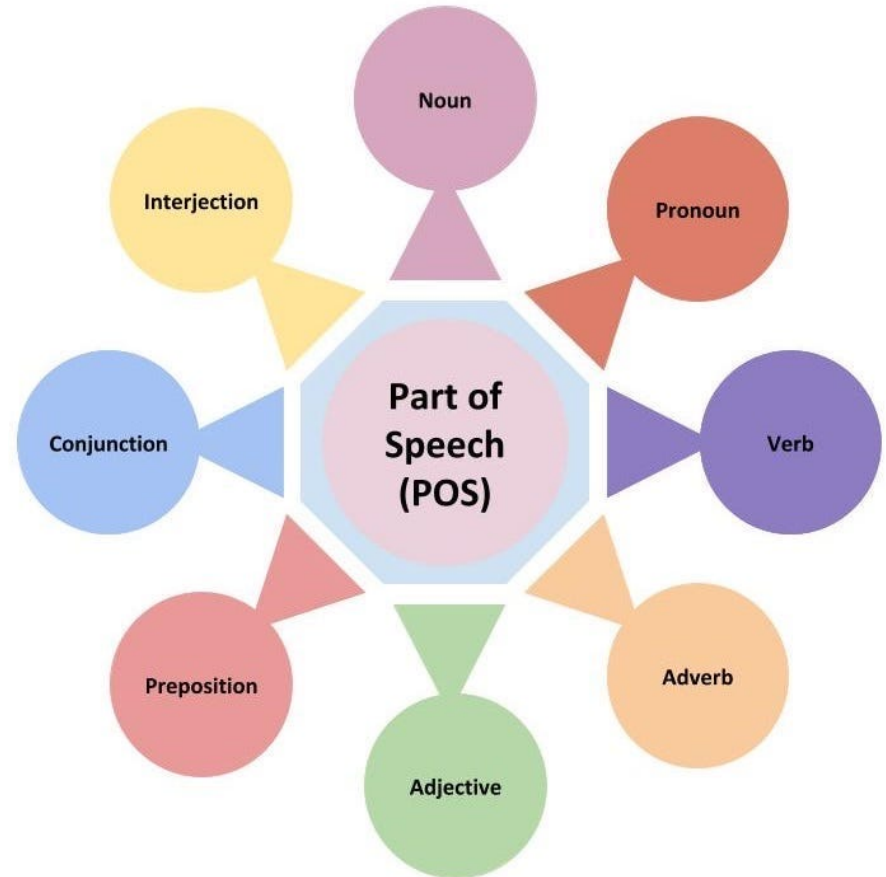
The/DT cat/NN sat/VBD on/IN the/DT mat/NN

Fort/NNP Worth/NNP is/VBZ in/IN Texas/NNP

The/DT old/NN man/VB the/DT boat/NN

PARTS OF SPEECH

- Different words have different functions
- Closed class: fixed membership, **function words**
 - e.g. prepositions (*in, on, of*), determiners (*the, a*)
- Open class: New words get added frequently
 - e.g. nouns (Twitter, Facebook), verbs (google), adjectives, adverbs



PENN TREE BANK TAG SET

45 Tags

Tag	Description	Example	Tag	Description	Example	Tag	Description	Example
CC	coordinating conjunction	<i>and, but, or</i>	PDT	predeterminer	<i>all, both</i>	VBP	verb non-3sg present	<i>eat</i>
CD	cardinal number	<i>one, two</i>	POS	possessive ending	<i>'s</i>	VBZ	verb 3sg pres	<i>eats</i>
DT	determiner	<i>a, the</i>	PRP	personal pronoun	<i>I, you, he</i>	WDT	wh-determ.	<i>which, that</i>
EX	existential 'there'	<i>there</i>	PRP\$	possess. pronoun	<i>your, one's</i>	WP	wh-pronoun	<i>what, who</i>
FW	foreign word	<i>mea culpa</i>	RB	adverb	<i>quickly</i>	WP\$	wh-possess.	<i>whose</i>
IN	preposition/ subordin-conj	<i>of, in, by</i>	RBR	comparative adverb	<i>faster</i>	WRB	wh-adverb	<i>how, where</i>
JJ	adjective	<i>yellow</i>	RBS	superlatv. adverb	<i>fastest</i>	\$	dollar sign	<i>\$</i>
JJR	comparative adj	<i>bigger</i>	RP	particle	<i>up, off</i>	#	pound sign	<i>#</i>
JJS	superlative adj	<i>wildest</i>	SYM	symbol	<i>+, %, &</i>	“	left quote	<i>' or “</i>
LS	list item marker	<i>1, 2, One</i>	TO	“to”	<i>to</i>	”	right quote	<i>' or ”</i>
MD	modal	<i>can, should</i>	UH	interjection	<i>ah, oops</i>	(left paren	<i>[, (, {, <</i>
NN	sing or mass noun	<i>llama</i>	VB	verb base form	<i>eat</i>)	right paren	<i>],), }, ></i>
NNS	noun, plural	<i>llamas</i>	VBD	verb past tense	<i>ate</i>	,	comma	<i>,</i>
NNP	proper noun, sing.	<i>IBM</i>	VBG	verb gerund	<i>eating</i>	.	sent-end punc	<i>. ! ?</i>
NNPS	proper noun, plu.	<i>Carolinas</i>	VBN	verb past part.	<i>eaten</i>	:	sent-mid punc	<i>: ; ... - -</i>

(Marcus et al., 1993)

Other corpora: Brown, WSJ, Switchboard

PART OF SPEECH TAGGING

- A disambiguation task: each word may have different senses/functions
 - The/DT **man/NN** bought/VBD a/DT boat/NN
 - The/DT old/NN **man/VB** the/DT boat/NN
- Some words have MANY functions:

earnings growth took a **back/JJ** seat

a small building in the **back/NN**

a clear majority of senators **back/VBP** the bill

Dave began to **back/VB** toward the door

enable the country to buy **back/RP** about debt

I was twenty-one **back/RB** then

A SIMPLE BASELINE

- Most words are easy to disambiguate
- **Most frequency class:** assign each word (token) its most frequently used class in the training set. (e.g., man/NN)
- Accuracy: 92.34% on the Wall Street Journal (WSJ) dataset!
- State of the art: ~ 97%
- Average English sentence: ~ 14 words
 - Sentence level accuracy: $0.92^{14} = 31\%$ vs $0.97^{14} = 65\%$
- POS tagging not solved yet!

HIDDEN MARKOV MODELS

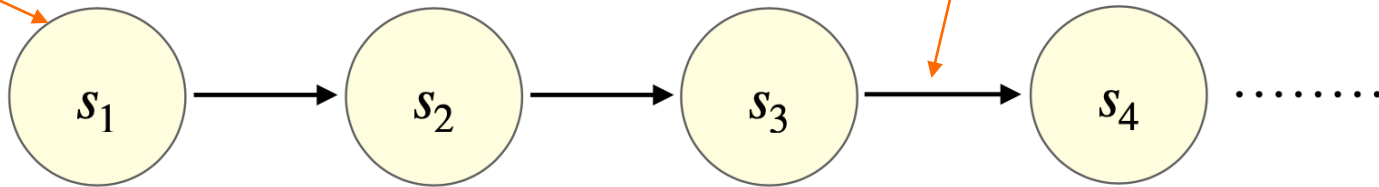
SOME OBSERVATIONS

- The function (or POS) of a word depends on its context
 - The/DT old/NN man/VB the/DT boat/NN
 - The/DT old/JJ man/NN bought/VBD the/DT boat/NN
- Certain POS combinations are extremely unlikely
 - $\langle JJ, DT \rangle$ or $\langle DT, IN \rangle$
- Better to make decisions on entire sequences instead of individual words (**Sequence modeling!**)

MARKOV CHAINS

$\Pi(s_1)$: initial prob. dist.

$P(s_t | s_{t-1})$: transitional prob.

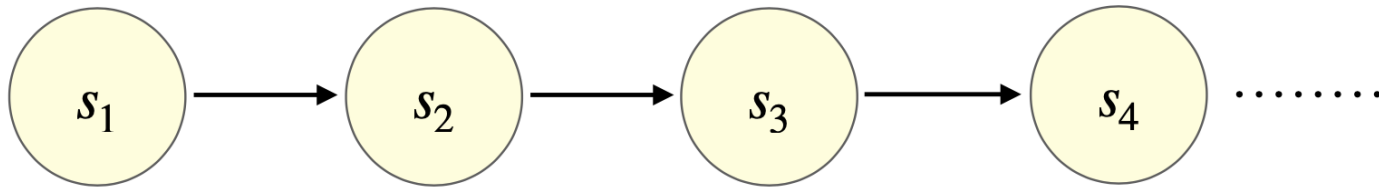


- Model probabilities of sequences of variables
- Each state can take one of K values ($\{1, 2, \dots, K\}$ for simplicity)
- Markov assumption:

$$P(s_t | s_{<t}) \approx P(s_t | s_{t-1})$$

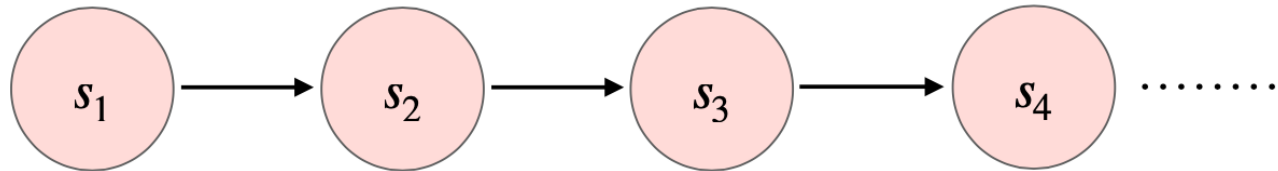
- Where have we seen this before?

MARKOV CHAINS



The/DT cat/NN sat/VBD on/IN the/DT mat/NN

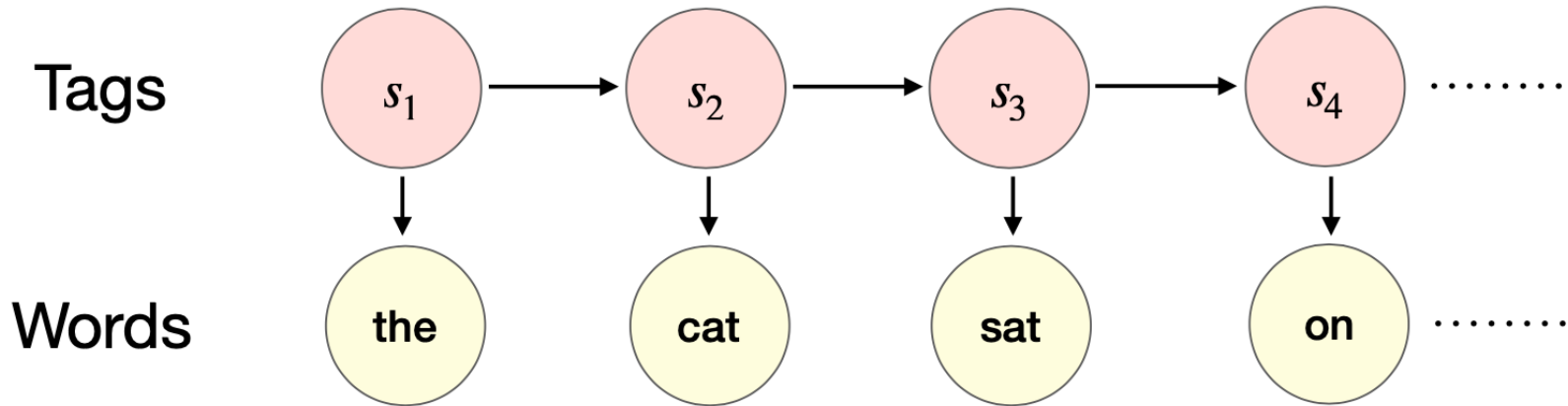
MARKOV CHAINS



The/?? cat/?? sat/?? on/?? the/?? mat/??

- We don't know the tags in the corpus.

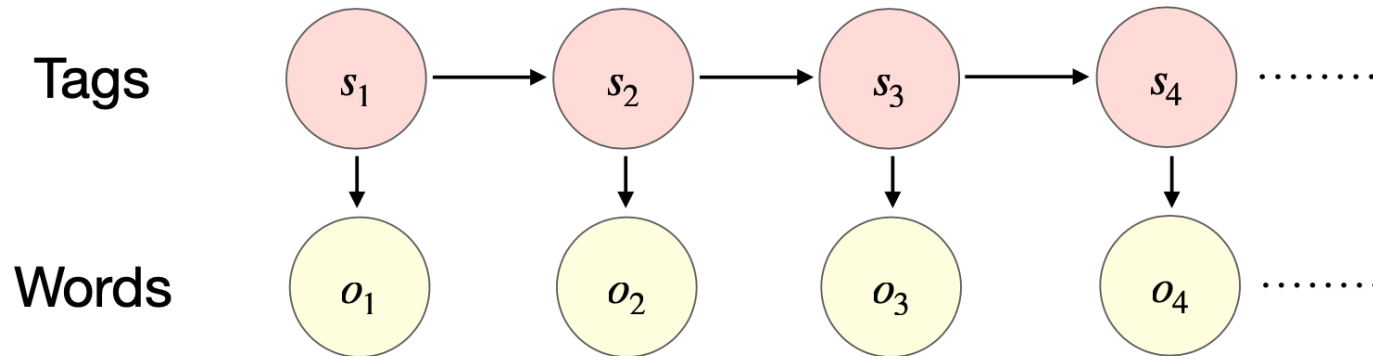
MARKOV CHAINS



The/?? cat/?? sat/?? on/?? the/?? mat/??

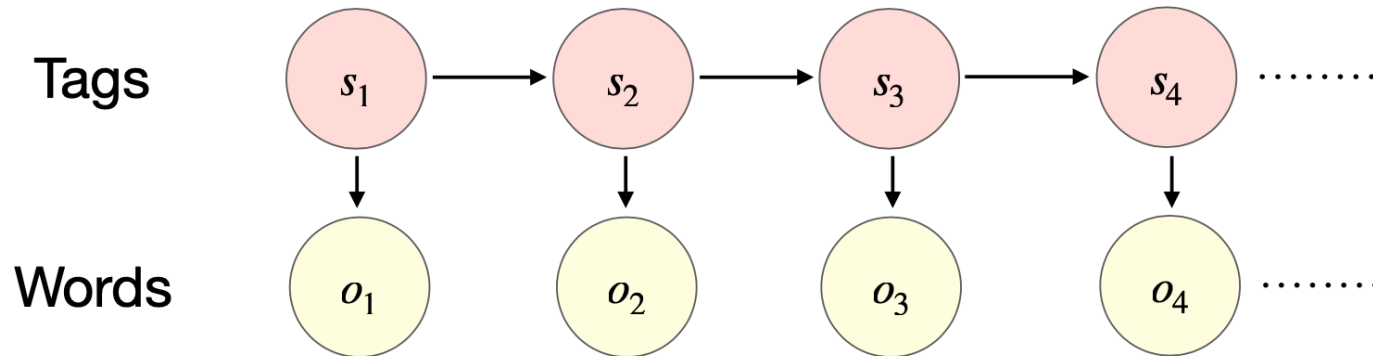
- We don't know the tags in the corpus.
- But we do observe the words!
- HMM allows us to jointly reason over both hidden and observed events.

COMPONENTS OF AN HMM



1. Set of **states** $S = \{1, 2, \dots, K\}$ and **observations** O
2. Initial state probability distribution: $\Pi(s_1)$
3. Transition probabilities: $P(s_{t+1} \mid s_t)$
4. Emission probabilities: $P(o_t \mid s_t)$

ASSUMPTIONS



1. Markov assumption:

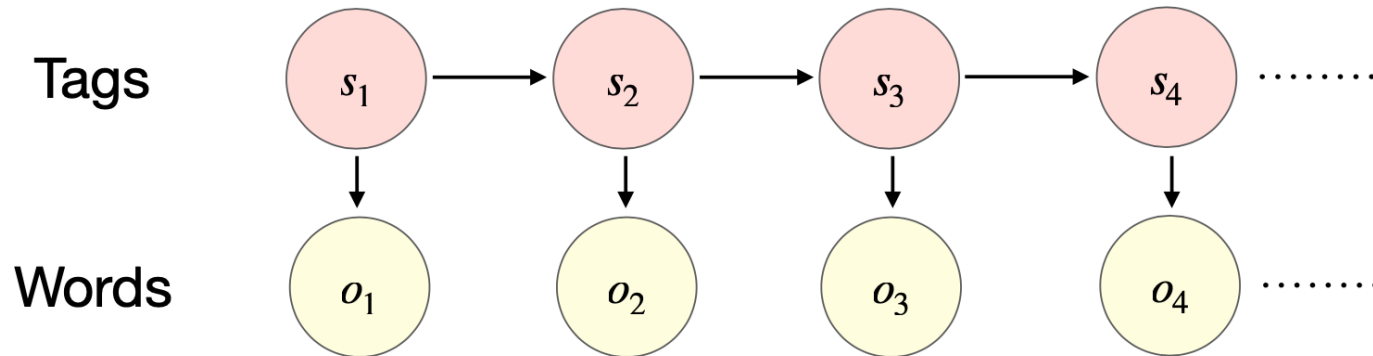
$$P(s_{t+1} \mid s_1, \dots, s_t) = P(s_{t+1} \mid s_t)$$

2. Output independence assumption:

$$P(o_t \mid s_1, \dots, s_t) = P(o_t \mid s_t)$$

Quiz: Which one of the two assumptions is stronger, and why?

SEQUENCE LIKELIHOOD



$$\begin{aligned} P(S, O) &= P(s_1, s_2, \dots, s_n, o_1, o_2, \dots, o_n) \\ &= \Pi(s_1)P(o_1|s_1) \prod_{i=2}^n P(s_i, o_i|s_{i-1}) \\ &= \Pi(s_1)P(o_1|s_1) \prod_{i=2}^n P(s_i|s_{i-1})P(o_i|s_i) \end{aligned}$$

LEARNING

○ Training Set:

1 Pierre/**NNP** Vinken/**NNP** ,/, 61/**CD** years/**NNS** old/**JJ** and/**CC** chairman/**NN** of/**IN** the/**DT** board/**NN** as/**IN** of/**IN** Nov./**NNP** 29/**CD** ./.

2 Mr./**NNP** Vinken/**NNP** is/**VBZ** chairman/**NN** of/**IN** Elsevier/**NNP** N.V./**NNP** ,/, the/**DT** Dutch/**NNP** publishing/**VBG** group/**NN** ./.

3 Rudolph/**NNP** Agnew/**NNP** ,/, 55/**CD** years/**NNS** old/**JJ** and/**CC** chairman/**NN** of/**IN** Consolidated/**NNP** Gold/**NNP** Fields/**NNP** PLC/**NNP** ,/, was/**VBD** named/**VBN** a/**DT** nonexecutive/**JJ** director/**NN** of/**IN** this/**DT** British/**JJ** industrial/**JJ** conglomerate/**NN** ./.

...

38,219 It/**PRP** is/**VBZ** also/**RB** pulling/**VBG** 20/**CD** people/**NNS** out/**IN** of/**IN** Puerto/**NNP** Rico/**NNP** ,/, who/**WP** were/**VBD** helping/**VBG** Hurricane/**NNP** Hugo/**NNP** victims/**NNS** ,/, and/**CC** sending/**VBG** them/**PRP** to/**TO** San/**NNP** Francisco/**NNP** instead/**RB** ./.

Maximum likelihood estimate:

Transition prob: $P(s_i|s_j) = \frac{c(s_i, s_j)}{c(s_j)}$

Emission Prob: $P(o|s) = \frac{c(s, o)}{c(s)}$

EXAMPLE: POS TAGGING

the/?? cat/?? sat/?? on/?? the/?? mat/??

$$\pi(DT) = 0.8$$

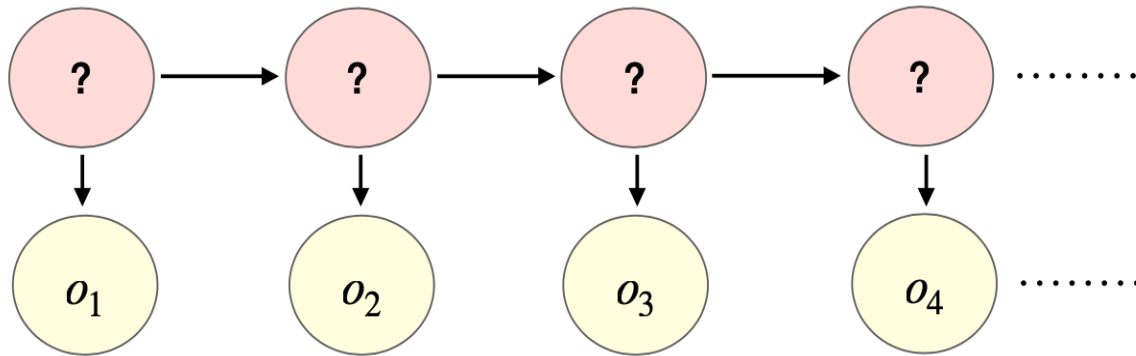
 s_{t+1}
 o_t

	DT	NN	IN	VBD
DT	0.5	0.8	0.05	0.1
NN	0.05	0.2	0.15	0.6
IN	0.5	0.2	0.05	0.25
VBD	0.3	0.3	0.3	0.1

	the	cat	sat	on	mat
DT	0.5	0	0	0	0
NN	0.01	0.2	0.01	0.01	0.2
IN	0	0	0	0.4	0
VBD	0	0.01	0.1	0.01	0.01

$$P(\text{The/DT, cat/NN, sat/VBD, on/IN, the/DT, mat/NN}) = 1.84 * 10^{-5}$$

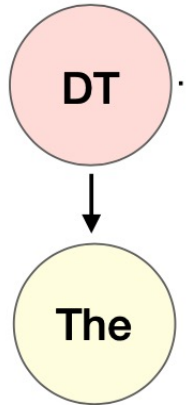
DECODING WITH HMMs



- **Task:** Find the most probable sequence of states $\langle s_1, s_2, \dots, s_n \rangle$, given the observations $\langle o_1, o_2, \dots, o_n \rangle$

$$\begin{aligned}\hat{S} &= \underset{S}{\operatorname{argmax}} P(S|O) = \underset{S}{\operatorname{argmax}} \frac{P(S)P(O|S)}{P(O)} \leftarrow \text{constant} \\ &= \underset{S}{\operatorname{argmax}} P(S)P(O|S) \\ &= \underset{S}{\operatorname{argmax}} \prod_{i=1}^n P(s_i|s_{i-1})P(o_i|s_i) \\ &\quad \begin{array}{l} \nearrow \text{transition} \qquad \nearrow \text{emission} \end{array}\end{aligned}$$

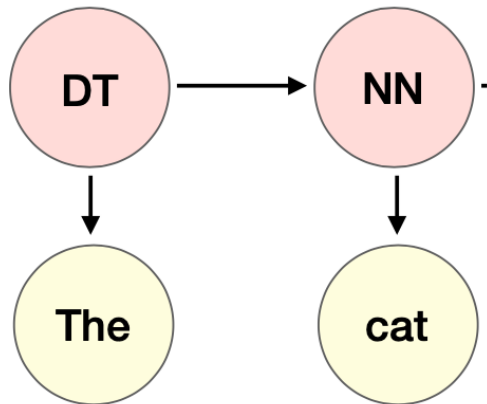
GREEDY DECODING



$$\underset{s}{\operatorname{argmax}} \Pi(s_1 = s)P(\text{The} | s) = \text{'DT'}$$

$$\hat{S} = \underset{s}{\operatorname{argmax}} \prod_{i=1}^n P(s_i | s_{i-1})P(o_i | s_i)$$

GREEDY DECODING

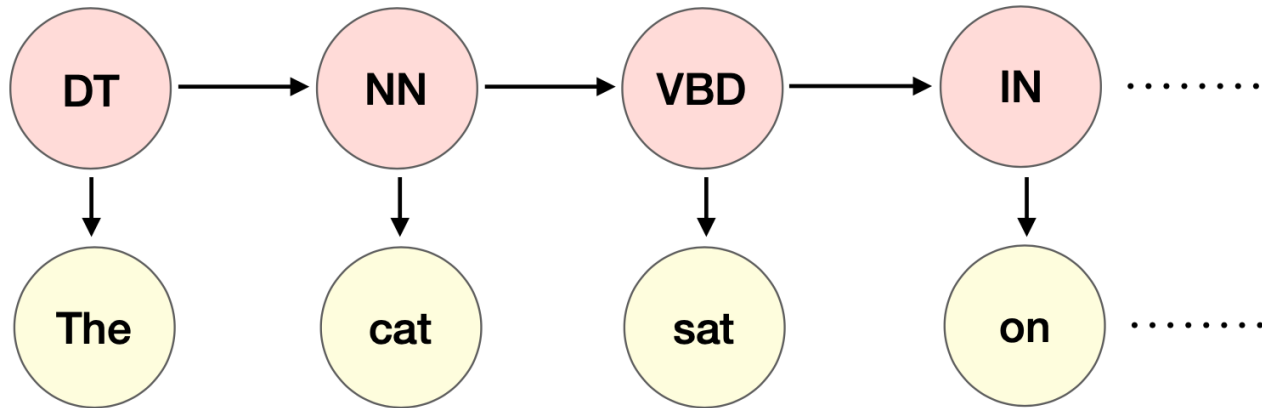


$$\underset{s}{\operatorname{argmax}} \Pi(s_1 = s)P(\textit{The} | s) = \textit{'DT'}$$

$$\underset{s}{\operatorname{argmax}} P(s_2 = s | \textit{DT})P(\textit{cat} | s) = \textit{'NN'}$$

$$\hat{S} = \underset{s}{\operatorname{argmax}} \prod_{i=1}^n P(s_i | s_{i-1})P(o_i | s_i)$$

GREEDY DECODING



$$\underset{s}{\operatorname{argmax}} \Pi(s_1 = s)P(\textit{The} | s) = \textit{'DT'}$$

$$\underset{s}{\operatorname{argmax}} P(s_2 = s | \textit{DT})P(\textit{cat} | s) = \textit{'NN'}$$

$$\forall i, \hat{s}_{i+1} = \underset{s}{\operatorname{argmax}} P(s | \hat{s}_i)P(o_{i+1} | s)$$

Not guaranteed to be optimal: local decision only!

VITERBI DECODING

- Use dynamic programming!
- Probability lattice, $M[T, K]$
 - T : Number of time steps
 - K : Number of states
- $M[i, j]$: Most probable sequence of states ending with state j at time i

VITERBI DECODING

DT

$$M[1,DT] = \pi(DT) P(\mathbf{the} | DT)$$

NN

$$M[1,NN] = \pi(NN) P(\mathbf{the} | NN)$$

VBD

$$M[1,VBD] = \pi(VBD) P(\mathbf{the} | VBD)$$

IN

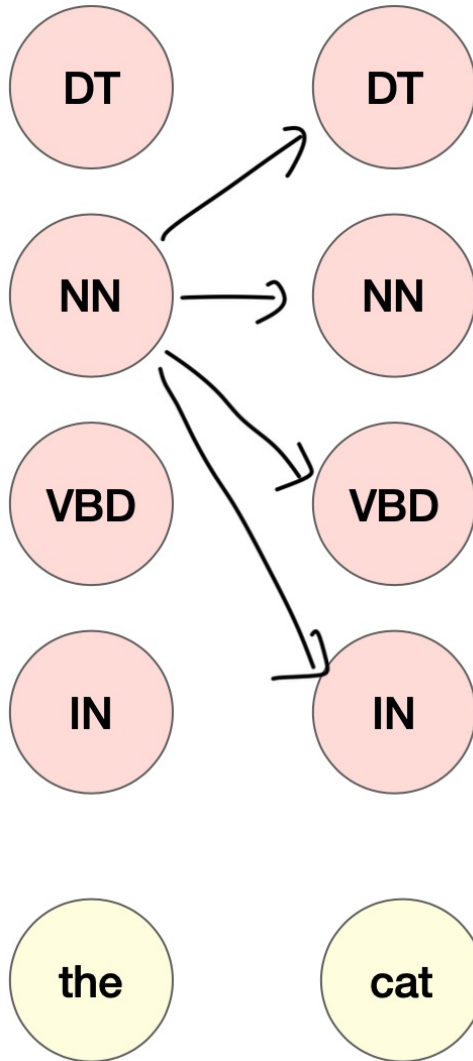
$$M[1,IN] = \pi(IN) P(\mathbf{the} | IN)$$

the

Forward →

VITERBI DECODING

suppose
 $k=NN$



$$M[2,DT] = \max_k M[1,k] P(DT|k) P(\mathbf{cat}|DT)$$

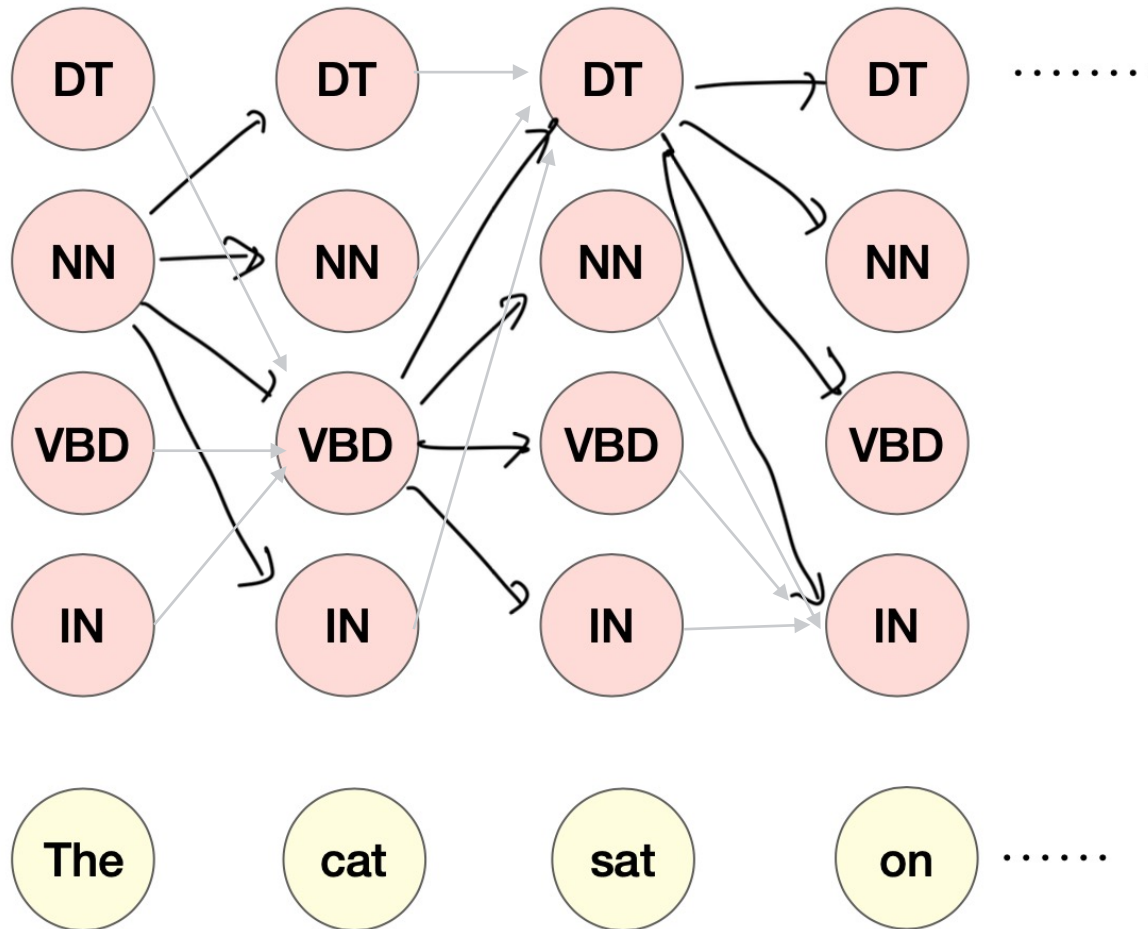
$$M[2,NN] = \max_k M[1,k] P(NN|k) P(\mathbf{cat}|NN)$$

$$M[2,VBD] = \max_k M[1,k] P(VBD|k) P(\mathbf{cat}|VBD)$$

$$M[2,IN] = \max_k M[1,k] P(IN|k) P(\mathbf{cat}|IN)$$

Forward \rightarrow

VITERBI DECODING



This is a recursive process!

Viterbi Algorithm needs to backtrack.

$$M[i, j] = \max_k M[i - 1, k] P(s_j | s_k) P(o_i | s_j) \quad 1 \leq k \leq K, 1 \leq i \leq N$$

QUIZ: VITERBI ALGORITHM

Assume

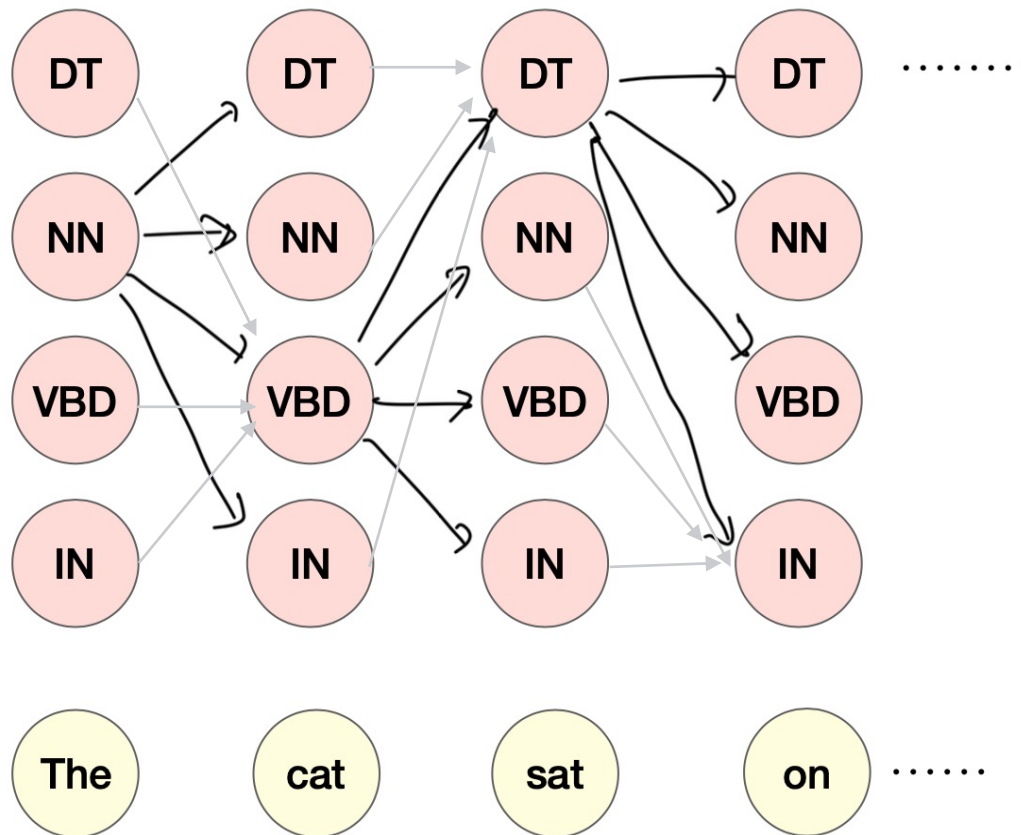
T : Number of time steps (sequence length)

K : Number of states

What is the time complexity of the Viterbi algorithm (in Big O)?

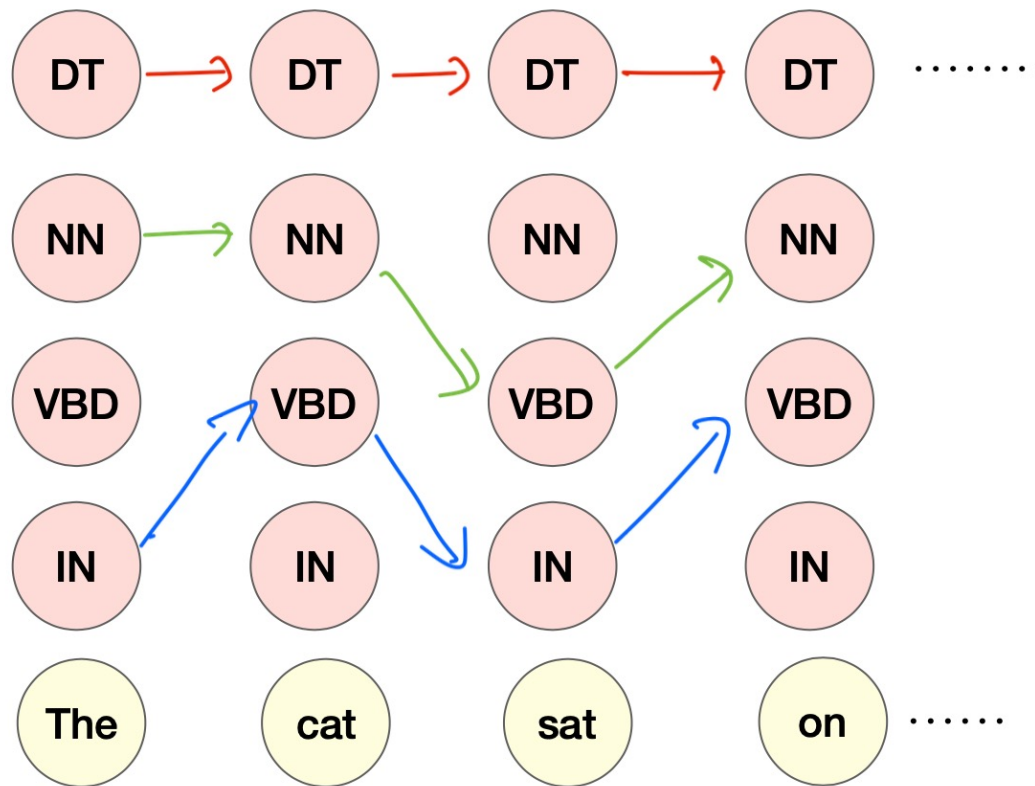
BEAM SEARCH

- When K (the number of states) is large, Viterbi algorithm is very expensive!



BEAM SEARCH

- But any paths have very low likelihood!



BEAM SEARCH

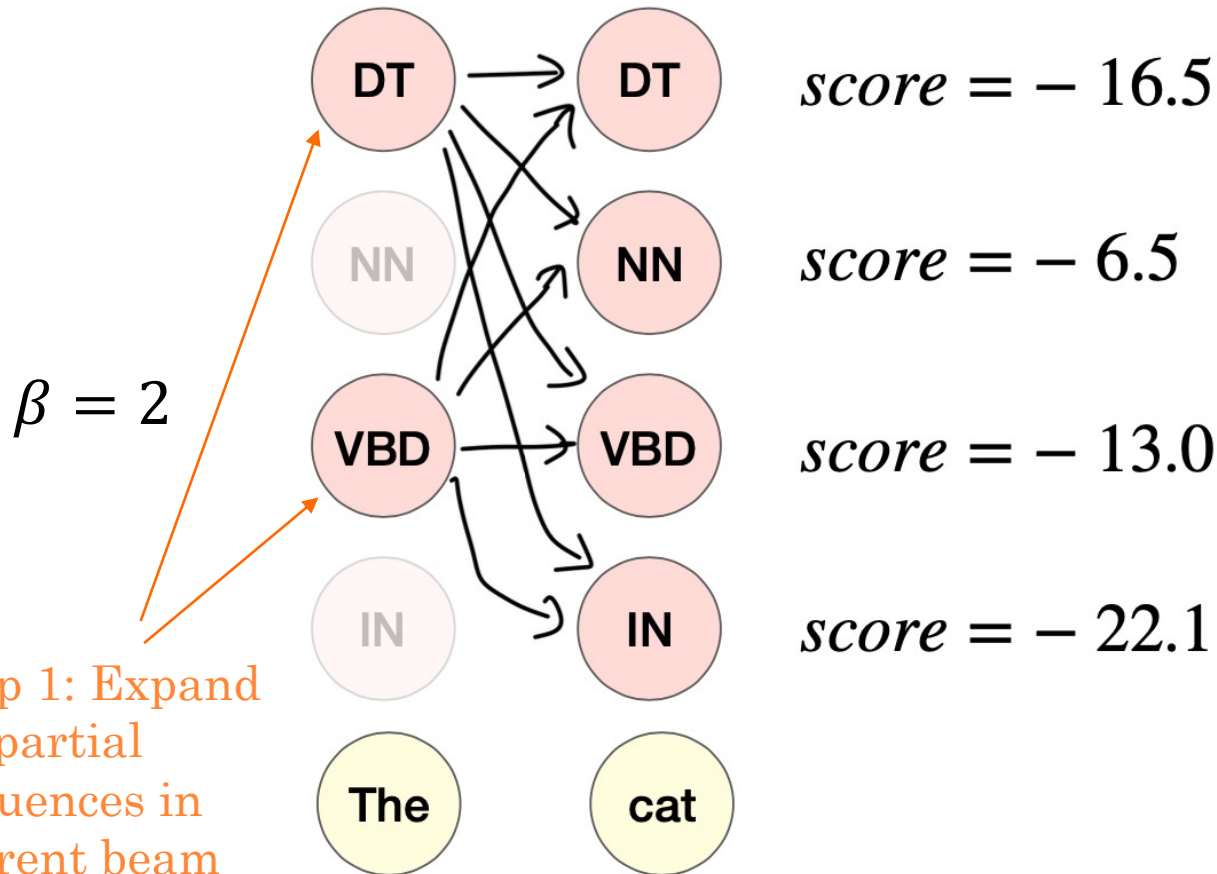
- Keep a fix number β of hypotheses at each stage:

$\beta = 2$

DT	<i>score</i> = - 4.1
NN	<i>score</i> = - 9.8
VBD	<i>score</i> = - 6.7
IN	<i>score</i> = - 10.1
The	

BEAM SEARCH

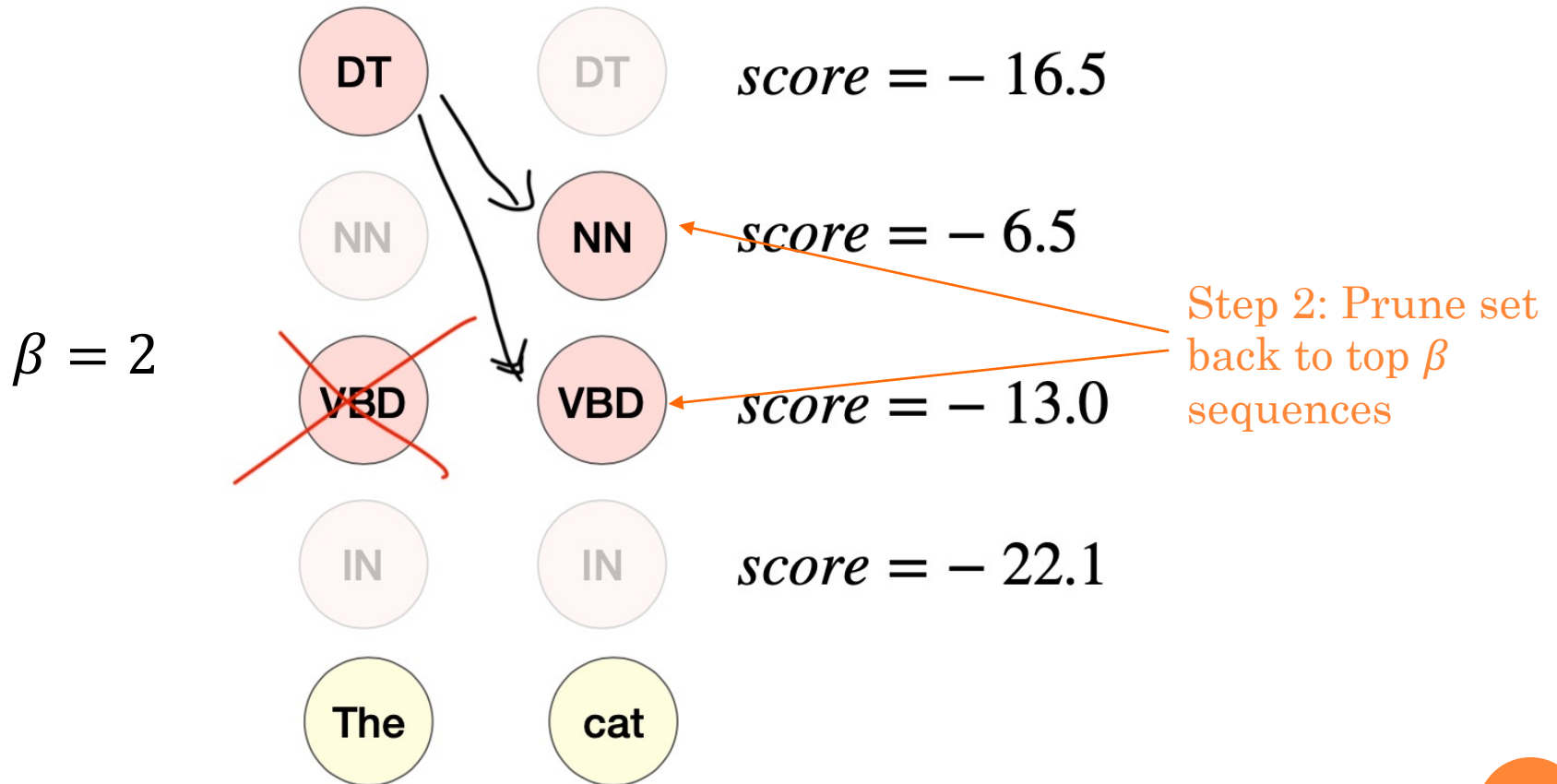
- Keep a fix number β of hypotheses at each stage:



Step 1: Expand all partial sequences in current beam

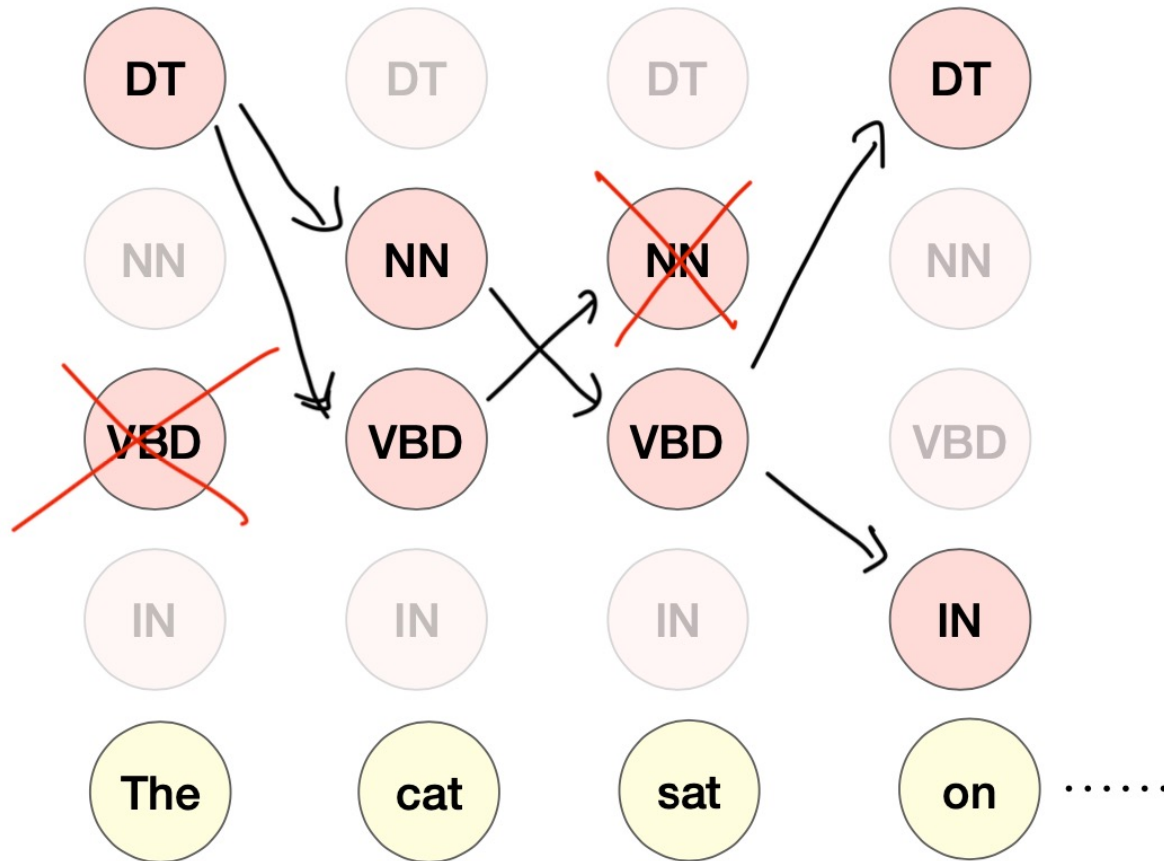
BEAM SEARCH

- Keep a fix number β of hypotheses at each stage:



BEAM SEARCH

- Keep a fix number β of hypotheses at each stage:



Step n : Pick $\max_k M[n, k]$ from within the beam and backtrack

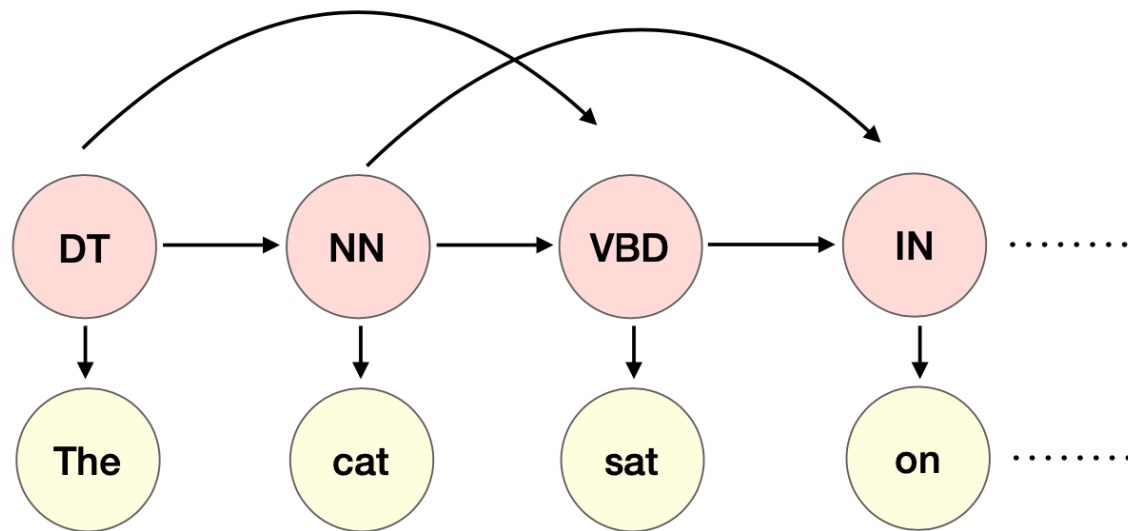
BEAM SEARCH

- If K (number of states) is too large, Viterbi algorithm is too expensive!
- Keep a fixed number of hypotheses at each stage
- Beam width β
- Trade-off (some) accuracy for efficiency

Quiz: What is the time complexity of Beam Search Viterbi Algorithm, given sequence length T , number of states K , and β ?

BEYOND BIGRAMS

- Real-world HMM taggers have more relaxed assumptions.
- Tri-gram HMM: $P(s_{t+1}|s_1, s_2, \dots, s_t) = P(s_{t+1}|s_{t-1}, s_t)$



Pros?

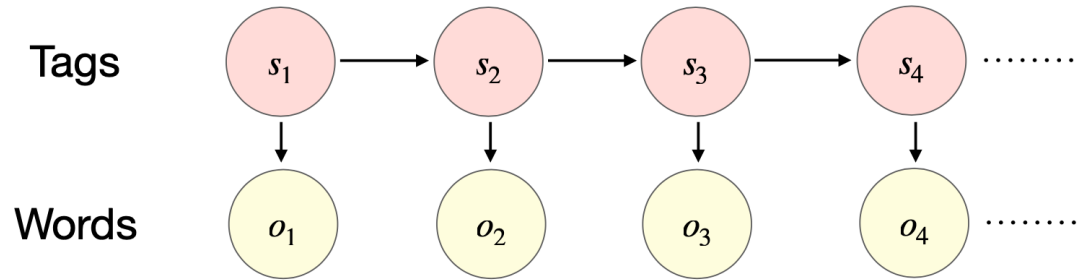
Cons?

LIMITATIONS OF HMM

- HMM is a generative model: $P(O | S)$
- Unknown (OOV) words happen often
- HMM relies on a fixed vocabulary (fixed-size emission probability matrix)
- Can't add arbitrary features easily
- Remember log-linear models (LR) can combine arbitrary models?
- But LR is is not a sequential model
- Enter the *Conditional Random Field!*
 - Discriminative model: $P(S | O)$

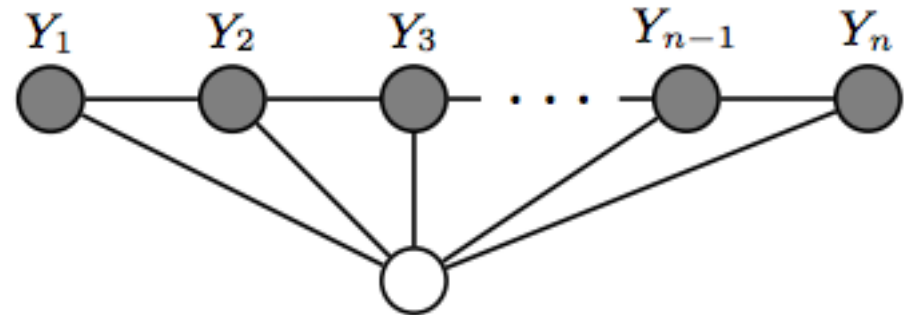
LINEAR CHAIN CRF

- HMM:



$$\hat{S} = \underset{S}{\operatorname{argmax}} \prod_{i=1}^n P(s_i | s_{i-1}) P(o_i | s_i)$$

- Linear chain CRF:



$$\hat{Y} = \underset{Y \in \mathcal{Y}}{\operatorname{argmax}} P(Y | X)$$

$$\mathbf{X} = X_1, \dots, X_{n-1}, X_n$$

\mathcal{y} is the set of all possible tag sequences

LINEAR CHAIN CRF

- Assigns a probability of the entire tag sequence Y , out of all possible sequences \mathcal{y} .
- A giant version of **multinomial logistic regression** for a single token.

$$p(Y|X) = \frac{\exp\left(\sum_{k=1}^K w_k F_k(X, Y)\right)}{\sum_{Y' \in \mathcal{y}} \exp\left(\sum_{k=1}^K w_k F_k(X, Y')\right)}$$

- F_k is the k^{th} feature function mapping $X \rightarrow Y$
- K is total number of features

LINEAR CHAIN CRF

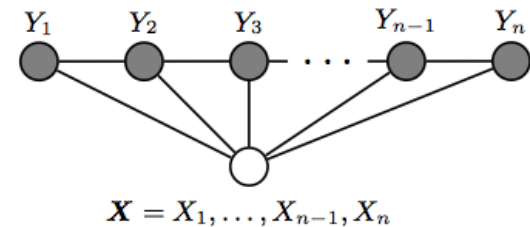
- Rename the denominator as a function $Z(X)$:

$$p(Y|X) = \frac{1}{Z(X)} \exp \left(\sum_{k=1}^K w_k F_k(X, Y) \right)$$

$$Z(X) = \sum_{Y' \in \mathcal{Y}} \exp \left(\sum_{k=1}^K w_k F_k(X, Y') \right)$$

- Global feature $F_k(X, Y)$ can be decomposed into a sequence of local features, where n is the length of the token sequence:

$$F_k(X, Y) = \sum_{i=1}^n f_k(y_{i-1}, y_i, X, i)$$



Sum: no
directions

linear chain! But no directions!

FEATURES IN LINEAR CHAIN CRF

- Discriminative models allow for many features
- Each feature f_k depends on any info from

$$(y_{i-1}, y_i, X, i)$$

- Example features:

$$\mathbb{1}\{x_i = \textit{the}, y_i = \text{DET}\}$$

$$\mathbb{1}\{y_i = \text{PROPN}, x_{i+1} = \textit{Street}, y_{i-1} = \text{NUM}\}$$

$$\mathbb{1}\{y_i = \text{VERB}, y_{i-1} = \text{AUX}\}$$

- Use feature template to extract features for each position i :

$$\langle y_i, x_i \rangle, \langle y_i, y_{i-1} \rangle, \langle y_i, x_{i-1}, x_{i+2} \rangle$$