CSE 4392 Special Topics
Natural Language Processing

Language Models

2024 Spring

An Example

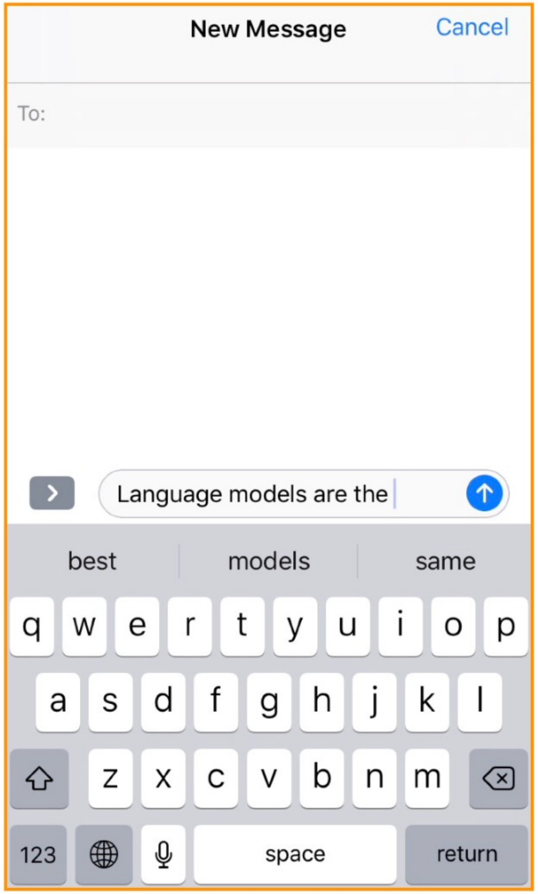
Today in Arlington, TX, it's 45F and sunny.

vs.

Today in Arlington, TX, it's 45F and blue.

- Both are grammatical
- But which is more likely?

Language Models are Everywhere



And Many Applications

- Predicting words is important in many situations
  - Machine translation

    P(a **smooth** finish) > *P*(a **flat** finish)
  - Speech recognition/Spell checking

    P(high school **principal**) > *P*(high school **principle**)
  - Information extraction, question answering

Impact on Downstream Applications

Language Resources	Adaptation	Word Cor.	Word Acc.	PP
1. Doc-A		54.5%	45.1%	49972
2. Trans-C(L)		63.3%	50.6%	1856.5
3. Trans-B(L)		70.2%	60.3%	318.4
4. Trans-A(S)		70.4%	59.3%	442.3
5. Trans-B(L)+Trans-A(S)	CM	72.6%	63.9%	225.1
6. Trans-B(L)+Doc-A	KW	72.1%	64.2%	247.5
7. Trans-B(L)+Doc-A	KP	73.1%	65.6%	259.7
8. Trans-A(L)		75.2%	67.3%	148.6

(Miki et al. 2006)

New Approach to Language Modeling Reduces Speech Recognition Errors by Up to 15%

Ankur Gandhe

Principal, Applied Scientist

Alexa Speech group, Amazon

What is a Language Model?

- Probabilistic model of a sequence of words.
  - How likely is a given phrase/sentence/paragraph/ document?

- Joint distribution:

$$P(w_1, w_2, \ldots, w_\mathrm{n})$$

Chain Rule

$$
\begin{aligned}
P(X_1, X_2, \ldots X_n) &= P(X_1)P(X_2|X_1)P(X_3|X_1, X_2)\ldots \\
&= \prod_{i=1}^{n} P(X_i|X_1, \ldots, X_{i-1})
\end{aligned}
$$

- Sentence: “the sun rises and shines”

P(the sun rises and shines) = P(the) * P(sun | the) * P(rises | the sun) * P(and | the sun rises) * P(shines | the sun rises and)

ESTIMATING THE PROBABILITIES

$$P(rises \mid the\ sun) = \frac{count(the\ sun\ rises)}{count(the\ sun)}$$

$$P(and \mid the\ sun\ rises) = \frac{count(the\ sun\ rises\ and)}{count(the\ sun\ rises)}$$

$$\vdots$$

Maximum Likelihood Estimate (MLE)

- With a vocabulary of size V,
  - number of sequences of length $n = V^n$
- Typical vocab size of 40k words (English):
  - even just considering sentences of <=11 words results in $4*10^{50}$ different sentences (number of atoms on earth only $\sim 10^{50}$)
- Use a corpus to count these word sequences

Markov Assumption

- Use only recent past in the sequence to predict next word

- Reduce the number of estimated parameters in exchange for model capacity (can model longer sentences now!)

- 1st order:
  $$P(shines|the\ sun\ rises\ and) \cong P(shines|and)$$
- 2nd order:
  $$P(shines|the\ sun\ rises\ and) \cong P(shines|rises\ and)$$

K-th Order Markov Chain

- Consider only the last k words from the context:

$$P(w_i \mid w_1 w_2 \ldots w_{i-1}) \approx P(w_i \mid w_{i-k} \ldots w_{i-1})$$

which implies the probability of a sequence is:

$$P(w_1 w_2 \ldots w_n) \approx \prod_i P(w_i \mid w_{i-k} \ldots w_{i-1})$$

k+1 gram

N-gram Language Models

- Unigram

$$P(w_1, w_2, ...w_n) = \prod_{i=1}^{n} P(w_i)$$

- Bigram

$$P(w_1, w_2, ...w_n) = \prod_{i=1}^{n} P(w_i|w_{i-1})$$

- And trigram, 4-gram, etc.
- Larger the n, more accurate and better the language model (but at a higher cost)
- Remember the data is *infinite*!

Text Generations using N-grams

Unigram *release millions See ABC accurate President of Joe Will cheat them a CNN megynkelly experience @ these word out- the*

Bigram *Thank you believe that @ ABC news, New Hampshire tonight and the false editorial I think the great people Nikki Haley . ''*

Trigram *We are going to MAKE AMERICA GREAT AGAIN! #MakeAmericaGreatAgain https: //t.co/DjkdAzT3WV*

$$\arg \max_{(w_1, w_2, \ldots, w_n)} \Pi_{i=1}^{n} P(w_i | w_{<i})$$

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Trigram *We are going to MAKE AMERICA GREAT AGAIN! #MakeAmericaGreatAgain https: //t.co/DjkdAzT3WV*

Typical LMs are not sufficient to handle long-range dependencies:

"Alice/Bob could not go to work that day because she/he had a doctor's appointment"

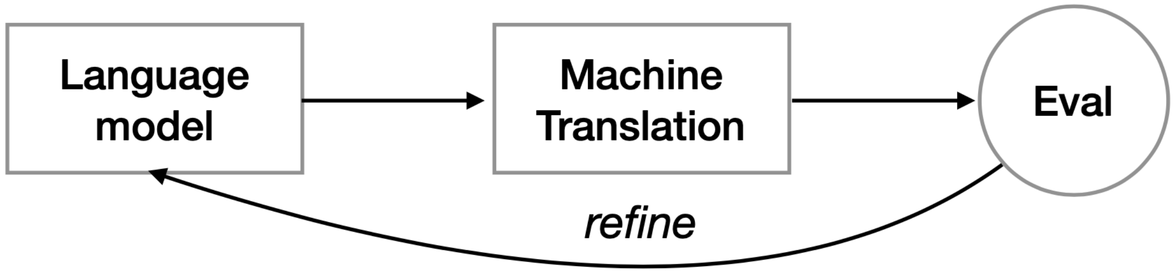
Evaluating Language Models

- A good language model should assign higher probability to typical, grammatically correct sentences

- Research process:
  - Train parameters on a suitable training corpus
    - Assumption: observed sentences ~ good sentences
  - Test on *different, unseen* corpus
    - Training on any part of test set not acceptable!
  - Evaluation metric

Extrinsic Evaluation

- Train LM → Apply to task → Observe accuracy



- Directly optimized for downstream tasks
  - Higher accuracy → better model

- Expensive, time consuming
- Hard to optimize downstream objective (indirect feedback)

Perplexity (per word)

- Measures how well a probability distribution (or a model) predicts a sample
- For a corpus S with sentences S_1, S_2, … S_n.

$$\text{ppl}(S) = 2^x \text{ where } x = -\frac{1}{W}\sum_{i=1}^{n} \log_2 P(S_i)$$

A form of cross entropy

where W is the total number of words in test corpus

- Unigram model: $$x = -\frac{1}{W}\sum_{i=1}^{n}\sum_{j=1}^{m} log_2 P(w_j^i)$$

j^{th} word in i^{th} sentence

- Minimizing perplexity ~ maximizing probability

Intuition of Perplexity

- If our n-gram model (with vocabulary V) has the following probability:

$$P(w_i|w_{i-n}, ...w_{i-1}) = \frac{1}{|V|} \quad \forall w_i$$

what i s?

$$\text{ppl} = 2^{-\frac{1}{W} W * log(1/|V|)} = |V|$$

- The model is "fine" with observing any word at every step!

Pros and Cons of Perlexity

Pros	Cons
Fast to compute, eliminate ”bad” models that can’t perform well in expensive real-world testing	Not good for final evaluation: measures model’s confidence, not accuracy
Model’s uncertainty/information density is useful information	Not fair comparison across models trained on different datasets
Statistically robust (not easily influenced by a single outlier sentence in the dataset)	Can reward models trained on toxic or outdated dataset

QUIZ: PPL OF BIGRAMS

- Given the following training corpus:

  S1: you have five apples

  S2: you have no oranges

  S3: no apples have you

- What is the ppl of the bigram language model on this test sentence:

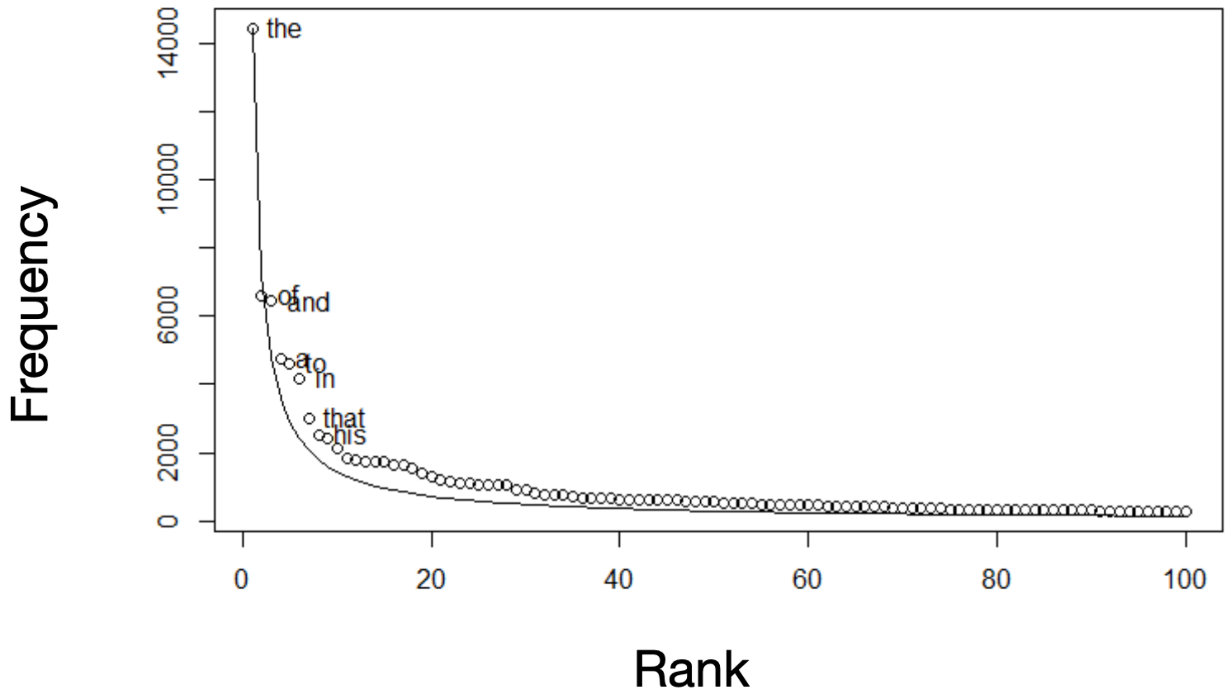
  S4: you have no apples

$$\text{ppl(S)} = 2^x \text{ where } x = -\frac{1}{W}\sum_{i=1}^{n}\log_2 P(S_i)$$

Generalization of N-grams

- Not all n-grams are observed in training data!
- Test corpus may contain some n-grams with zero probability under our model
  - Training data: *Google News*
  - Test data: *Shakespeare*
  - $P\,(affray \mid voice\ doth\ us) = 0 \rightarrow P\,(\text{test set}) = 0$
  - Undefined perplexity

Sparsity in Languages



$$freq \propto \frac{1}{rank}$$

Zipf's Law

- Long tail of infrequent words
- Most finite-size corpora will have this problem

Smoothing

- Handling sparcity by making sure every probability is non-zero in our models
  - Additive: Add a small amount to all probabilities
  - Discounting: Redistribute probability mass from observed n-grams to unobserved ones
  - Back-off: Use lower order n-grams if higher ones are too sparse
  - Interpolation: Use a combination of different granularities of n-grams

Intuition of Smoothing

- When we have sparse statistics:

  P(w | denied the)

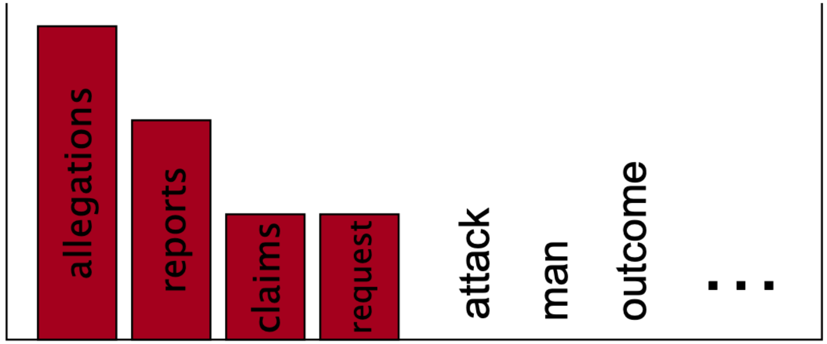
  3 allegations

  2 reports

  1 claims

  1 request

  7 Total



- Steal probability mass to generalize better:

  P(w | denied the)

  2.5 allegations

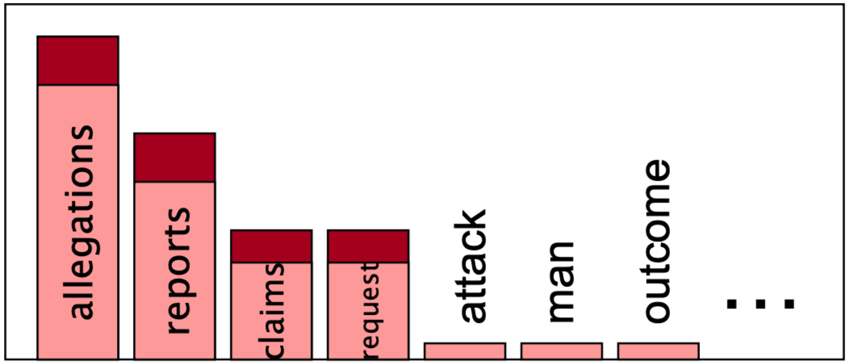
  1.5 reports

  0.5 claims

  0.5 request

  2 others

  7 Total



Laplace Smoothing

- Also known as add-alpha
- Simplest form of smoothing: just add a small alpha to all counts and renormalize!
- Max likelihood for bigrams:

$$P(w_i|w_{i-1}) = \frac{C(w_{i-1}, w_i)}{C(w_{i-1})}$$

- After smoothing:

$$P(w_i|w_{i-1}) = \frac{C(w_{i-1}, w_i) + \propto}{C(w_{i-1}) + \propto |V|}$$

Raw Bigram Counts (Berkeley restaurant corpus)

- Out of 9222 sentences

w_i

w_{i-1}

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

The numbers in the table are $c(w_{i-1}\ w_i)$

Credits: Dan Jurafsky)

Smoothed Bigram Counts

- Alpha = 1 in this case:

	i	want	to	eat	chinese	food	lunch	spend
i	6	828	1	10	1	1	1	3
want	3	1	609	2	7	7	6	2
to	3	1	5	687	3	1	7	212
eat	1	1	3	1	17	3	43	1
chinese	2	1	1	1	1	83	2	1
food	16	1	16	1	2	5	1	1
lunch	3	1	1	1	1	2	1	1
spend	2	1	2	1	1	1	1	1

Credits: Dan Jurafsky)

Smoothed Bigram Probabilities

- Alpha = 1 in this case:

$$P^*(w_n|w_{n-1}) = \frac{C(w_{n-1}w_n)+1}{C(w_{n-1})+V}$$

	i	want	to	eat	chinese	food	lunch	spend
i	0.0015	0.21	0.00025	0.0025	0.00025	0.00025	0.00025	0.00075
want	0.0013	0.00042	0.26	0.00084	0.0029	0.0029	0.0025	0.00084
to	0.00078	0.00026	0.0013	0.18	0.00078	0.00026	0.0018	0.055
eat	0.00046	0.00046	0.0014	0.00046	0.0078	0.0014	0.02	0.00046
chinese	0.0012	0.00062	0.00062	0.00062	0.00062	0.052	0.0012	0.00062
food	0.0063	0.00039	0.0063	0.00039	0.00079	0.002	0.00039	0.00039
lunch	0.0017	0.00056	0.00056	0.00056	0.00056	0.0011	0.00056	0.00056
spend	0.0012	0.00058	0.0012	0.00058	0.00058	0.00058	0.00058	0.00058

Credits: Dan Jurafsky)

Problem with Laplace Smoothing

raw counts

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

reconstituted counts

	i	want	to	eat	chinese	food	lunch	spend
i	3.8	527	0.64	6.4	0.64	0.64	0.64	1.9
want	1.2	0.39	238	0.78	2.7	2.7	2.3	0.78
to	1.9	0.63	3.1	430	1.9	0.63	4.4	133
eat	0.34	0.34	1	0.34	5.8	1	15	0.34
chinese	0.2	0.098	0.098	0.098	0.098	8.2	0.2	0.098
food	6.9	0.43	6.9	0.43	0.86	2.2	0.43	0.43
lunch	0.57	0.19	0.19	0.19	0.19	0.38	0.19	0.19
spend	0.32	0.16	0.32	0.16	0.16	0.16	0.16	0.16

$$c^*(w_{n-1}w_n) = \frac{[C(w_{n-1}w_n)+1] \times C(w_{n-1})}{C(w_{n-1})+V}$$

QUIZ

$$c^*(w_{n-1}w_n) = \frac{[C(w_{n-1}w_n)+1] \times C(w_{n-1})}{C(w_{n-1})+V}$$

- Given the following training corpus:

  S1: you have five apples

  S2: you have no oranges

  S3: no apples have you

- Produce the bigram raw counts table and reconstituted counts table using alpha = 1:

	you	have	five	apples	no	oranges
you						
have						
five						
apples						
no						
oranges						

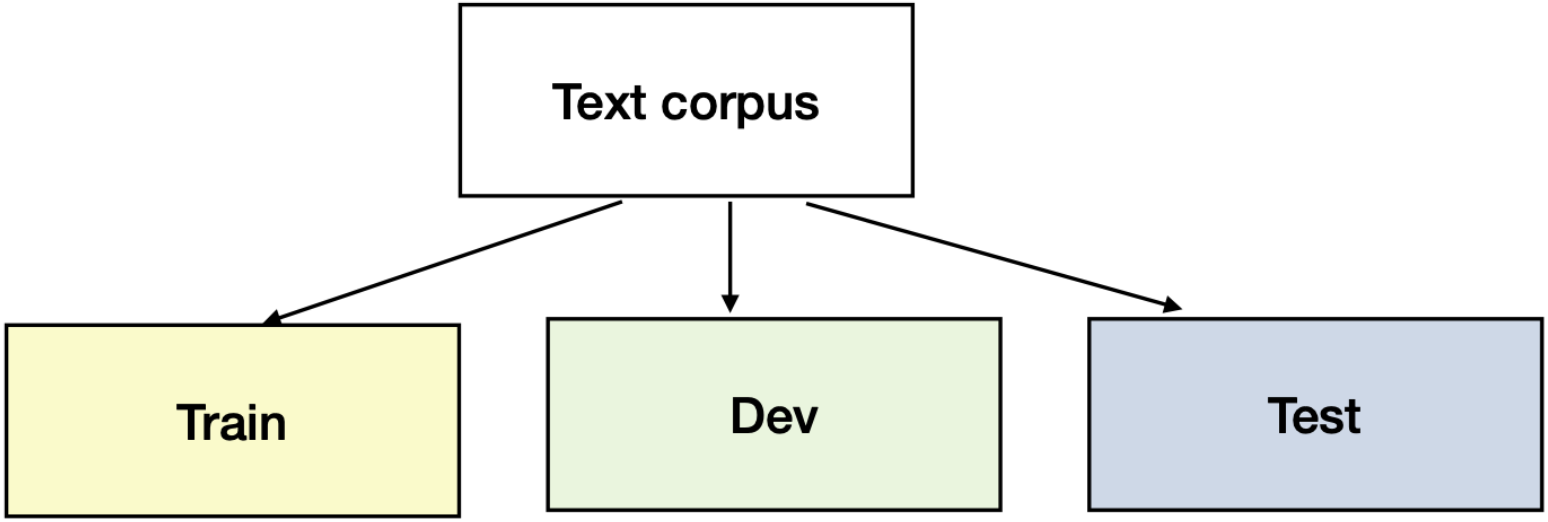
Linear Interpolation

$$\hat{P}(w_i|w_{i-1},w_{i-2}) = \lambda_1 P(w_i|w_{i-1},w_{i-2}) + \lambda_2 P(w_i|w_{i-1}) + \lambda_3 P(w_i)$$

$$\sum_i \lambda_i = 1$$

- Use a combination of models to estimate probability
- Strong empirical performance

Choosing Lambdas



- First, estimate n-gram prob. on training set
- Then, estimate lambdas (*hyperparameters*) to maximize probability on the held-out dev set

AVERAGE-COUNT (CHEN & GOODMAN, 1998)

$$P_{\text{interp}}(w_i | w_{i-n+1}^{i-1}) = \lambda_{w_{i-n+1}^{i-1}} P_{\text{ML}}(w_i | w_{i-n+1}^{i-1}) + (1 - \lambda_{w_{i-n+1}^{i-1}}) P_{\text{interp}}(w_i | w_{i-n+2}^{i-1})$$

Recursive definition!

- Like simple interpolation, but with more specific lambdas, $\lambda_{w_{i-n+1}^{i-1}}$ conditioned on the context.
- Partition $\lambda_{w_{i-n+1}^{i-1}}$ according to average number of counts per non-zero element: $\frac{c(w_{i-n+1}^{i-1})}{|w_i : c(w_{i-n+1}^{i}) > 0|}$
- for denser estimates of n-gram probabilities

Intuition for Average-Count

- Case 1: C (on the mat) = 10, C(on the cat) = 10, C(on the rat) = 10, C(on the bat) = 10, …
- Case 2: C (on the mat) = 40, C(on the cat) = 0, C (on the rat) = 0, C(on the bat) = 0, …
- Which provides a better estimate for P(mat | on the)?
- Larger weights on non-sparse estimates
- What if C (the mat) = 37, C(the cat) = 1, C (the rat) = 1, C(the bat) = 1, … ?

DISCOUNTING

Bigram count in training	Bigram count in heldout set
0	.0000270
1	0.448
2	1.25
3	2.24
4	3.23
5	4.21
6	5.23
7	6.21
8	7.21
9	8.26

- Determine some "mass" to remove from probability estimates
- Redistribute mass among unseen n-grams
- Just choose an absolute value *d* to discount:

$$P_{AbsDiscount}(w_i|w_{i-1}) = \frac{C(w_{i-1}w_i) - d}{C(w_i)} + \lambda(w_{i-1})P(w_i)$$

the difference is roughly 0.75, hence $d = 0.75$

Back-off

- Use n-gram if enough evidence, else back off to (n-1)-gram

$$P_{bo}(w_i \mid w_{i-n+1} \cdots w_{i-1})$$

$$= \begin{cases} d_{w_{i-n+1} \cdots w_i} \dfrac{C(w_{i-n+1} \cdots w_{i-1} w_i)}{C(w_{i-n+1} \cdots w_{i-1})} & \text{if } C(w_{i-n+1} \cdots w_i) > k \\ \alpha_{w_{i-n+1} \cdots w_{i-1}} P_{bo}(w_i \mid w_{i-n+2} \cdots w_{i-1}) & \text{otherwise} \end{cases}$$

- d = amount of discounting
- α = back-off weight

INTERPOLATON VS BACKOFF

- To determine the probability of n-grams with *zero* counts:
  - Both use the distributions of lower-order n-grams

- To determining the probability of n-grams with *nonzero* counts:
  - Interpolation uses the distribution of lower-order n-grams
  - Backoff does not.

Other Language Models

- Discriminative models:
  - train n-gram probabilities to directly maximize performance on end task (e.g., as feature weights)
- Parsing-based models
  - handle syntactic/grammatical dependencies
- Topic models (word distributions for topics not sequences)