CSE 3302/5307 Programming Language Concepts

Homework6 - Fall 2023

Due Date: Sep.30, 2024, 11:59p.m. Central Time

Problem1 - 30%

We've seen how to define natural numbers using church encoding in untyped lambda calculus:

$$\begin{aligned}\mathbf{0} &= \lambda f.\lambda x.\ x\\ \mathbf{1} &= \lambda f.\lambda x.\ f\ x\\ &\dots\\ \mathbf{n} &= \lambda f.\lambda x.\ f^n\ x\\ &\dots\end{aligned}$$

Note that church encoding cannot represent negative integers.

(a) Propose a **simple** method to extend church numerals to representation of integers. Give a concrete example for representation of integer **-5** with your proposed method. Hint: you may try to use pairs.

(b) Define the XOR function given two boolean inputs in lambda calculus and test that it works.

(c) Define a new multiplication operation *mulint* that works on the representation of integers you defined.

For the last two points, you can directly use basic logical functions defined in the lecture such as *not*, *and* and *or* as well as the *mul* you wrote for natural numbers in an earlier assignment as well as .

Problem2 - 30%

Given the definition of Fibonacci number

$$F_0 = 0, F_1 = 1, F_i = F_{i-1} + F_{i-2}$$

(a) Use *fix* to write a lambda function called *fib*: int $\rightarrow$ int to compute the n-th Fibonacci number.

(b) Test that your function works by showing detailed steps for $fib\,3$. Use the Z combinator defined in the lecture for fix and do not treat it as a black box.

Problem3 - 40%

Given the following λ expression:

```
let x = 2 in
  let y = 4 in
    let f1 = \x.\y.x+2*y in
      let f2 = \x.\y.2*x-y in
      f2 (f1 y x) 3
```

Using the environment model for lambda calculus with let,
(a) Define closures. (Be careful and refer to lecture slides);
(b) Show detailed multi-step evaluation process of the λ expression above.
The environment should be clearly shown in each step.

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