CSE 3302/5307 Programming Language Concepts

Homework4 - Fall 2023

Due Date: Sep.23, 2024, 11:59p.m. Central Time

> In the proofs for this assignment, failure to follow the step-justify proof format demonstrated in the lecture, or providing incorrect or missing "need to prove" statements, may result in significant loss of marks. This is to motivate you to write more correct proofs and seek help when needed.

Problem1 - 25%

Define the judgment $\Gamma \vdash e : t$ inductively and provide a natural language explanation for each inference rule. Additionally, for each inference rule, specify the corresponding expression form using the Backus-Naur form (BNF) of Simply-typed Lambda calculus.

Problem2 - 25%

Prove the **exchange lemma**: If $\Gamma, x : t_1, y : t_2, \Gamma' \vdash e : t$, then $\Gamma, y : t_2, x : t_1, \Gamma' \vdash e : t$. (proof by induction on derivation of $\Gamma, x : t_1, y : t_2, \Gamma' \vdash e : t$).

Problem3 - 25%

Prove the **weakening lemma**: If $\Gamma \vdash e : t$ then $\Gamma, x : t' \vdash e : t$ (provided x not in Dom(Γ)). Hint: you may need to use the Exchange lemma.

Problem4 - 25%

Prove the **substitution lemma**: If $\Gamma, x : t' \vdash e : t$ and $\Gamma \vdash v : t'$ then $\Gamma \vdash e[v/x] : t$.
Hint: you may need to use the Exchange and Weakening lemmas.

Name: _____ UTA ID: _____