CSE 3302/5307 Programming Language Concepts

Homework3 - Fall 2023

Due Date: Sep.16, 2023, 11:59p.m. Central Time

Problem1 - 20%

Evaluate $(\lambda x.\ ((\lambda y.\ x + z + 3)\ 3)5)$ using call-by-value and call-by-name. Show the complete steps of evaluation.

Problem2 - 20%

Given $Y = \lambda f.(\lambda x.f(x\,x))(\lambda x.f(x\,x))$, show that $g(Y\,g) = Y\,g$ by performing call-by-name evaluation on $Y\,g$.

Problem3 - 30%

Prove by induction: If $FV(e_1) = \emptyset$ and $e_1 \to e_2$, then $FV(e_2) = \emptyset$.

- Given the following definitions:

  1. Rules of free variables

$$\frac{}{FV(x) = \{x\}} \qquad \frac{FV(e_1) = S_1 \quad FV(e_2) = S_2}{FV(e_1\ e_2) = S_1 \cup S_2} \qquad \frac{FV(e) = S}{FV(x.e) = S - \{x\}}$$

  2. Judgment form: **define** $e_1 \to e_2$

$$\frac{}{(\lambda x.e)\ v \to e[v/x]} \qquad \frac{e_1 \to e_1'}{e_1\ e_2 \to e_1'\ e_2} \qquad \frac{e_2 \to e_2'}{v\ e_2 \to v\ e_2'}$$

- And given this lemma:

  Lemma 1. $FV(e_1[e_2/x]) \subseteq (FV(e_1) - \{x\}) \cup FV(e_2)$

By induction on derivation of $e_1 \to e_2$

1. Case $\frac{}{(\lambda x.e)\ v \to e[v/x]}$ Need to Prove:

2. Case $\frac{e_1 \to e_1'}{e_1\ e_2 \to e_1'\ e_2}$ Need to Prove:

3. Case $\frac{e_2 \to e_2'}{v\ e_2 \to v\ e_2'}$ Need to Prove:

Problem4 - 30%

Church numerals use lambdas to create a representation of numbers. They can represent natural numbers $\mathbf{0}, \mathbf{1}, \mathbf{2}, ...$, as follows:

$$\begin{aligned}
\mathbf{0} &= \lambda f.\lambda x.\ x \\
\mathbf{1} &= \lambda f.\lambda x.\ f\ x \\
\mathbf{2} &= \lambda f.\lambda x.\ f\ (f\ x) \\
\mathbf{3} &= \lambda f.\lambda x.\ f\ (f\ (f\ x)) \\
&\dots \\
\mathbf{n} &= \lambda f.\lambda x.\ f^n\ x \\
&\dots
\end{aligned}$$

Church numerals takes two parameters f and x. Church numerals n means apply f to x n times. You can read more about church numerals on the internet.

(a) Define addition in λ calculus, and then show the evaluation of $3 + 2$.

(b) Define multiplication in λ calculus (Hint: you can use definition of addition), and then show the evaluation of 3×2.