CSE 3302/5307 Programming Language Concepts

Homework 2 - Fall 2024

Due Date: Sep. 2, 2024, 11:59p.m. Central Time

Problem1 - 30%

(a) Consider looking at page 21 in slide "inductive-proof". In the proof of the second case $\frac{n\ nat}{S(n)\ nat}$, what is the assumption in this case and what is the difference between assumption and I.H.?

(b) We define a judgment form $add'\ n_1\ n_2\ n_3$ (another definition for addition):

$$\frac{}{add'\ Z\ Z\ Z}add'Z \qquad \frac{add'\ n_1\ n_2\ n_3}{add'\ (Sn_1)\ n_2\ (Sn_3)}add'l \qquad \frac{add'\ n_1\ n_2\ n_3}{add'\ n_1\ (Sn_2)\ (Sn_3)}add'r$$

For which rule we can use its inversion rule? If there exists such rule, point it out and give an explanation. If no rules can be inverted, give an explanation.

(c) We define a judgment form $IsNat\ x\ a$.

$$\frac{x\ nat}{IsNat\ x\ true}Nat \qquad \frac{x\ list}{IsNat\ x\ false}List$$

For which rule we can use its inversion rule? If there exists such rule, point it out and give an explanation. If no rules can be inverted, give an explanation.

Problem2 - 35%

(a) Give an inductive definition of the judgment form max n_1 n_2 n_3, which indicates the max number between n_1 and n_2 is n_3.
Hint: think of how we defined *add* by knowledge of *nat*.

(b) Prove by induction: if max n_1 n_2 n_3, then max n_2 n_1 n_3.

Problem3 - 35%

(a) Write the inductive definitions of len l n and append l_1 n l_2 and explain with natural language each rule in the inductive definition.

(b) Prove by induction: if len l n and append l n_1 l' then len l' (S n). Attempt the prove using induction on two different derivations.

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